

# Advanced Information Technology in Simulation Based Life Cycle Design

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## ABSTRACT

In this research a Collaborative Optimization (CO) approach for multidisciplinary systems design is used to develop a decision based design framework for non-deterministic optimization. To date CO strategies have been developed for use in application to deterministic systems design problems. In this research the decision based design (DBD) framework proposed by Hazelrigg (1996a, 1998) is modified for use in a collaborative optimization framework. The Hazelrigg framework as originally proposed provides a single level optimization strategy that combines engineering decisions with business decisions in a single level optimization. By transforming this framework for use in collaborative optimization one can decompose the business and engineering decision making processes. In the new multilevel framework of Decision Based Collaborative Optimization (DBCO) the business decisions are made at the system level. These business decisions result in a set of engineering performance targets that disciplinary engineering design teams seek to satisfy as part of subspace optimizations. The Decision Based Collaborative Optimization framework more accurately models the existing relationship between business and engineering in multidisciplinary systems design.

## NOMENCLATURE

$C_F$	fixed cost
$C_T$	total cost
$C_u$	unit cost
$C_V$	variable cost
$E(?)$	expected value
$F$	objective function in optimization problem
$NR$	net revenue
$P$	price
$PNR$	present value of net revenue
$a$	vector of attributes
$c_i$	demand coefficient for attribute $i$
$c_{price}$	demand coefficient for price
$d$	vector of compatibility (or discrepancy) constraints
$d_i$	compatibility constraint in discipline $i$
$g$	vector of constraints
$g_i$	vector of constraints in discipline $i$
$n_{cost}$	number of cost-related variables
$n_a$	number of attributes

$n_{ss}$	number of subspaces (disciplines)
$p$	vector of parameters
$(p_v)_i$	variable cost parameter associated with cost-related variable $i$
$q$	demand
$q_B$	baseline demand
$u$	utility
$x$	vector of design variables
$x_{aux}$	vector of auxiliary design variables
$(x_{aux})_{ij}$	vector of auxiliary design variables corresponding to $y_{ij}$
$x_i$	vector of local design variables for discipline $i$
$x_{sh}$	vector of shared design variables
$(x_{sh})_i$	vector of shared design variables used in discipline $i$
$x_{ssi}$	vector of subspace design variables in discipline $i$
$(x_{ss}^*)_i$	vector of optimal target values in discipline $i$
$x_{sys}$	vector of system level design variables
$(x_{sys}^*)_i$	vector of targets (i.e., system level design variables) sent down to discipline $i$
$y$	vector of state variables
$y_i$	vector of state variables calculated as output from discipline $i$
$y_{ij}$	vector of coupling variables calculated as output from discipline $i$ and used as input to discipline $j$
$(?)^o$	system level target variable
$(?)^*$	variable at the optimum design

## INTRODUCTION

Increasing attention has been paid to the notion that engineering design is a decision-making process (Chen, et. al, 1998, 2000, Kim, et. al, 2000, Azarm, 2000). This notion is consistent with the definition of decision as a choice from among a set of options and as an irrevocable allocation of resources. The approach of decision-based design (DBD) is built upon this notion. Rooted from more than two hundred years of research in the field of decision science, economics, operations research and other disciplines, decision-based design (DBD) provides a rigorous foundation for design, which enables engineers to identify the best trade-off and focus on where the payoffs are greatest.

Engineering design involves the generation of design alternatives or options and the selection of the best one. Since the number of possible design options is practically infinite for most products, human judgement is needed to decide which options to include in the consideration of alternative designs and

which to neglect. Moreover an appropriate value measure has to be determined in order to compare and rank order design options. It is impossible to know exactly how a particular design alternative will perform before it is built. However the product cannot be built until it is selected. Evidently engineers have to make their selection *a priori*, without full knowledge of the consequence of this certain selection. Thus design is always a matter of normative decision making under uncertainty and risk.

Optimization techniques have been widely applied to select the preferred design options from the set of alternatives taken into consideration without having to explicitly evaluate all possible alternatives in the set. There exists a close relationship between decision making and optimization. In general, decision making has three elements: generation or identification of options, assignment of expectations on each option and the application of preferences to determine the preferred choice. An optimization problem involves the maximization or minimization of an objective function or functional  $F(x)$  in the feasible region of design variables  $x$ . A careful comparison between decision making and optimization will reveal that options, expectations and preferences are all present in optimization. The option space is equivalent to the set of permissible values of  $x$  in the feasible region. The expectation of any given  $x$  is assigned by  $F(x)$ , and the preference is stated that more is better (maximization) or less is better (minimization). Thus optimization can be used to capture the properties of decision-making. This recognition allows the application of rigorous decision theory to the case of optimization.

## DECISION-BASED DESIGN (DBD) FRAMEWORK

Application of decision-based design within an optimization domain, requires practitioners to formulate valid objective functions for proper decision-making. Most of the research in the field of optimization has been focused on the solution of the optimization problem such as development, improvement and implementation of search methods to locate the optimum, while little attention has been paid to optimization problem formulation. In fact, the issue of problem formulation is of the same significance, if not more, as the issue of finding the optimal solution. A solution obtained using any search method is no better than the objective function chosen for the optimization. If an irrelevant objective is used, the solution is equally irrelevant (Hazelrigg 1996a, 1997). Therefore a primary concern in DBD is the development a mathematically sound objective function. Recognizing that design is a decision-making process, it is imperative to construct an objective function which is able to capture the preferences of rational decision makers in system design problems involving risk. The decision-based design (DBD) framework of Hazelrigg 1996a, 1998 (Fig. 1) provides a basis for exploring this issue.

## The Rule of Rational Decisions

The aforementioned decision-based design framework implements the concept of rational decisions. Rational decisions follow the rule that the preferred decision is the option whose expectation has the highest value. In a normative approach, decisions involve options, expectations and values. An expectation combines the possible outcomes of an alternative with probabilities of occurrence of each possible outcome. Note that in general expectations are not equivalent to outcomes. An outcome refers to what actually happens after a decision is made to select a certain option, while an expectation refers to what is expected to happen, based on available knowledge, as the result of a decision before the decision is made. In other words expectations are associated with what *will* happen, therefore, expectations relate to the future. In the process of engineering design, it is practically impossible to predict the future with precision and certainty. The outcomes of most options in the design option space cannot be determined with certainty prior to the decision to select one option. Hence expectations are always probabilistic. Engineers are forced to make decisions under uncertainty and risk.

Unlike problem-solving, decision-making cannot be conducted in the absence of human values. The purpose of values in decision making is to rank order alternatives according to the preference of decision makers. In the case of optimization, engineers seek a design which maximizes value, and the objective function serves as a numerical value function to automate the process of rank ordering. By means of an objective function, a real scalar is assigned to each design alternative in accordance with the decision maker's preference. In this sense an objective function serves as a utility function in the context of economics. Note that utilities are determined by

preferences and so is the objective function. A necessary condition for the existence of an objective function in decision-based design is that decision makers or design engineers are rational individuals whose preferences and indifferences between all pairs of outcomes in the design space exists and comprises a transitive set.

## Decision Making/Optimization Under Uncertainty

Due to the nature of engineering design, expectations on design alternatives can never be determined with certainty. Particular care should be taken in the formulation of an objective function when risk is present. It is imperative that the objective function (or utility function in the context of economics) must be valid under conditions of uncertainty and risk. The von Neumann-Morgenstern (vN-M) utility is such a value measure. Built upon the notion of von Neumann-Morgenstern lottery and six rigorous axioms (von Neumann, 1953), the normative framework for decision making under risk leads to a simple but profound result -- the so-called expected utility theorem: "The utility of a lottery is the sum of the utilities of all possible outcomes of the lottery weighted by their probabilities of occurrence." It follows that the preferred choice from among a set of risky options is the option with the highest expected utility. In the case of optimization, the axioms of vN-M utility should be adhered to and the objective function should entail the assignment of a vN-M utility to each design alternative under consideration.

## Characteristics of DBD Framework

The normative framework of decision-based design implements the concept of rational decisions. It facilitates vN-M utility as a measure of value against which design alternatives can be compared and optimal designs sought.

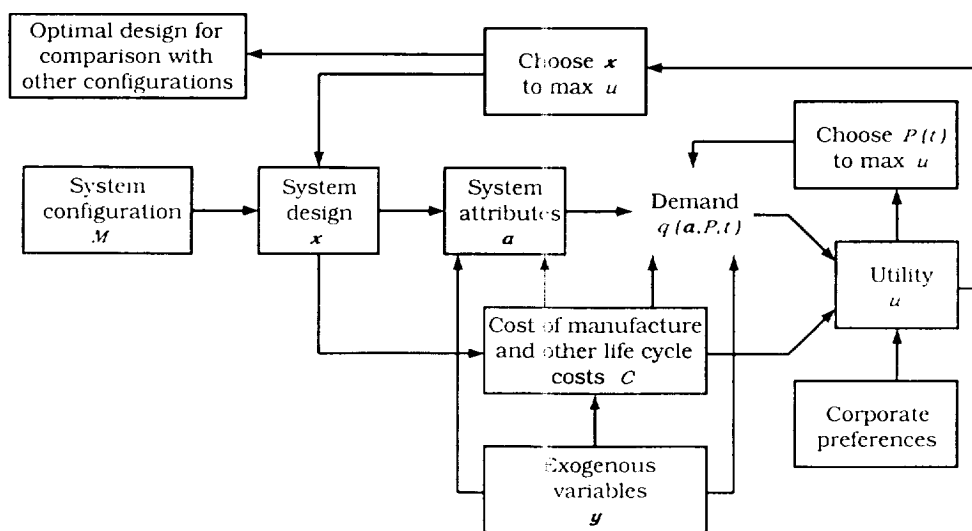


Figure 1. Decision-Based Design Framework (Hazelrigg, 1996b)

The DBD framework may view the objective of systems design as one of maximizing profit. Profit, which is also referred to as net revenue ( $NR$ ), is revenue generated by the product less all costs generated by the product. Revenue is the sum of products sold times their prices, in other words, it can be calculated as the product of the demand  $q$  and the price  $P$  at which the product is sold. Costs are the sum of things bought multiplied by their prices. Total cost  $C_T$  consists of: cost of manufacture, and all other life cycle costs such as costs of research, design, maintenance and repair and much more. The calculation of net revenue ( $NR$ ) can be summarized as Eq. (1):

$$NR = P \cdot q - C_T \quad (1)$$

To account for the time value of money since revenues and costs are spread over periods of time, a better index for profit would be the present value of net revenue ( $PNR$ ) which can be obtained from properly discounting net revenue ( $NR$ ) to the present and integrating it over time.

Recognizing most products are designed to make money, a valid objective function for optimization or decision making under uncertainty and risk can be established according to the rule of rational decisions: in the process of decision-based design, the optimizer should seek to maximize the expected vN-M utility of the profit, or net revenue ( $NR$ ).

The axioms of vN-M utility dictates that utilities are not arbitrarily selected. Rather, utilities must provide proper rank ordering of design alternatives in the case of vN-M lottery. In non-deterministic optimization where uncertainty and risk cannot be neglected, the decision maker's risk preference towards money must be taken into consideration. In general, the risk preference of the decision maker towards money leads to a utility of money that has diminishing marginal value (Bernoulli, 1738). Thus profit or net revenue ( $NR$ ) itself is not a valid utility under such circumstances.

## MULTIDISCIPLINARY ENTERPRISE MODEL

Design is inherently a multidisciplinary process. Traditionally multidisciplinary design has focused on disciplines within the field of engineering analysis, such as aerodynamics, solid mechanics, kinematics, control, and many others. In the approach of decision-based design, engineers are compelled to look at the design from a broader viewpoint. Indeed, decision-based design encompasses not only the engineering disciplines but also business disciplines including economics, marketing, operation research and more. In addition to aiming at improving the performance of a design, engineers must be aware of the substantial impact of non-engineering disciplines on the goal of design. Effective communications between engineers and experts in the business field are vital to produce successful designs.

The DBD framework of Hazelrigg (1996, 1998) combines both engineering and business performance simulations in a single level all-at-once optimization approach. In the current research the Hazelrigg framework (Fig. 1) has been decomposed into the multidisciplinary enterprise model shown in Fig. 2. The decomposed system consists of two major elements: the engineering disciplines and the business discipline. The work in engineering disciplines focuses on predicting the performance of the product for different design configurations, as well as satisfying performance targets set in the business discipline (i.e., management). The role of the business discipline centers on providing targets for performance improvements in order to yield higher profit. These two organizations are coupled through attributes  $a$ , total cost  $C_T$  and demand  $q$ .

Attributes  $a$  refer to the performances of a product. These performances directly influence customer demand. Examples of attributes include speed, acceleration, comfort, quality, reliability, safety, etc. Prediction of attributes  $a$  can be obtained from the engineering disciplines. Such information will be used as input into the business discipline to estimate the demand  $q$  for the product, which will then be used to determine the profit or net revenue. In order to compute profit, an estimate of the total cost  $C_T$  is required.  $C_T$  is again computed in the engineering disciplines. Since total cost is influenced by the number of product manufactured and/or sold, estimation of demand  $q$  is called for to feed back into the engineering disciplines. In this research it is assumed that the demand for the product, the amount of the product manufactured and the amount of the product sold are equal.

The prediction of product attributes  $a$  and total cost  $C_T$  is conducted in the engineering disciplines. The engineering disciplines module is partitioned into a module of product performance and a module of manufacturing cost and life cycle costs. The set of variables which uniquely defines a specific design are referred to as engineering design variables  $x$ . Engineers usually have control over these variables. The estimation of demand  $q$ , net revenue  $NR$  and, ultimately, the expected utility of net revenue, are managed in the business discipline. The price of the product not only directly affects the amount of profit or net revenue (see Eq. 1), it is also an important factor driving the demand  $q$ , which is also affected by the product attributes  $a$ . Mathematically speaking,  $q$  can be modeled as a function of attributes  $a$  and the price,  $P$ .

$$q = \bar{q}(a, P) \quad (2)$$

Note that price is free to be chosen by the decision maker, therefore it should be treated as a design variable.

In the context of multidisciplinary design, the term "system analysis (SA)" is often used to describe the process of predicting the performance of an engineering artifact using

numerical simulation, which is part of the multidisciplinary enterprise model investigated in this research. In fact, simulation based design tools can be used to assist in the analyses within the entire multidisciplinary system. Under these circumstances, the analysis of the enterprise model is no different than a “system analysis,” only of a larger scale. The research results of multidisciplinary design can be readily applied to the multidisciplinary enterprise model. Note that the equations of physical laws involved in the system analysis are sometimes referred to as “constraints” in the field of decision science.

The performance predictions obtained from a system analysis (SA) are referred to as states  $y$  in the context of multidisciplinary design. If we think of the multidisciplinary enterprise model (Fig. 2) as one big SA, then the attributes  $a$ , total cost  $C_T$ , demand  $q$  and net revenue  $NR$  are components of the system states. In this paper states  $y$  are used to represent the engineering performance predictions, calculated in the product performance module of the multidisciplinary enterprise model. Therefore, not all of the states  $y$  are attributes  $a$ . Only those states which influence product demand  $q$  will be included in the set of attributes  $a$ . In decision-based collaborative optimization framework (DBCO), the system level optimization treats attributes  $a$  as system level design variables  $x_{sys}$  which influence demand  $q$  and serve as targets for the discipline designers to satisfy.

Note that variability exists in the engineering design variables  $x$  and all simulation-based design tools employed in each discipline have prediction errors associated with them.

## SYSTEM OPTIMIZATION

The multidisciplinary enterprise model discussed above belongs to the type of so-called non-hierarchic systems (or coupled/networked systems). Such systems, usually of fairly large scale, are characterized by large numbers of design variables  $x$  and parameters  $p$ , large numbers of requirements or constraints  $g$ , and a high level of coupling between participating disciplines which are intrinsically linked to one another. Coupling results from the information exchange that are required within the system analysis. This occurs where the output of one discipline is required as input to another discipline and vice versa. Some of the mathematical terms, defined by Balling and Sobieszczanski-Sobieski (1994), are briefly reviewed below and will be used hereinafter.

In a coupled system the design variables  $x$  can be decomposed into the set of shared variables  $x_{sh}$  and  $n_{ss}$  sets of local variables  $x_i$ , where  $i$  ranges from 1 to  $n_{ss}$ , and  $n_{ss}$  is the total number of subspaces (or disciplines). The set of shared variables  $x_{sh}$  contains design variables that are needed by more than one discipline. The set of local variables  $x_i$  includes design variables associated with discipline  $i$  only. The set of  $x_{sh}$  and the sets of  $x_i$  are mutually exclusive subsets of the set of design

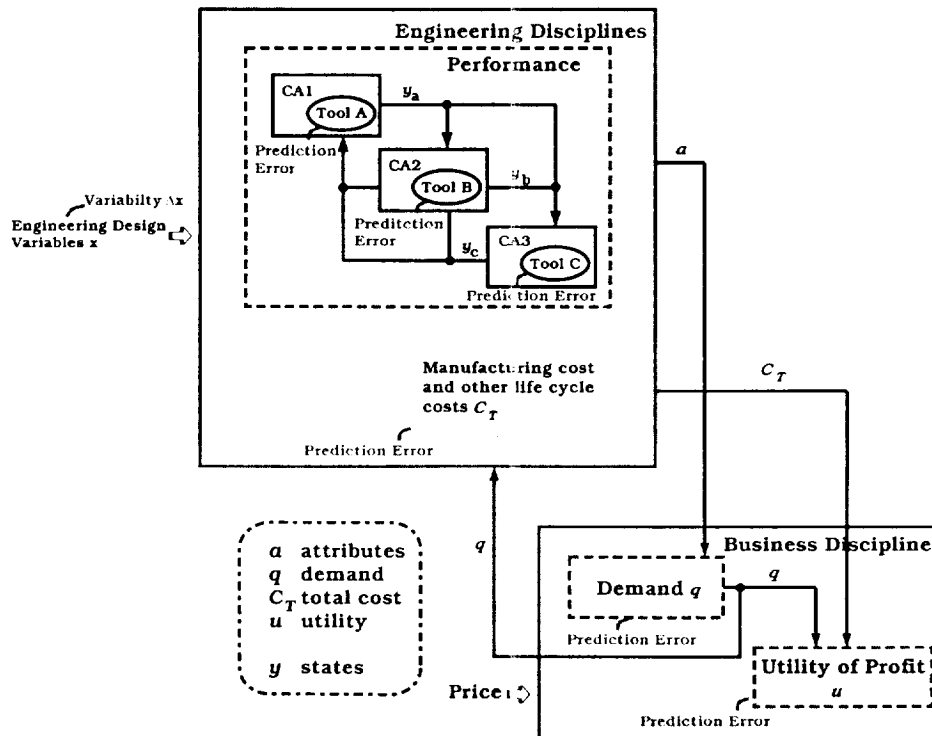


Figure 2. Multidisciplinary Enterprise Model

variables  $x$ . The term  $(x_{sh})_i$  is used to represent the shared design variables that are needed in discipline  $i$ .

The set of output from discipline  $i$  is denoted by  $y_i$ . The set of system states  $y$  is composed of all the  $y_i$ 's in the  $n_{ss}$  disciplines. The term  $y_{ij}$  is introduced to represent the output from discipline  $i$  which is also used as input in another discipline  $j$ . Note that  $y_{ij}$  are the subset of coupling variables in the set of  $y_i$ .

The vector  $g_i$  contains the design constraints associated with discipline  $i$ . It is assumed that no constraint will stretch over more than one discipline. It is also assumed that each inequality constraint has been formulated such that zero is the permissible value, and the constraint is satisfied when it is greater than or equal to zero. In conventional design optimization, constraints can be formulated to guard against system failure or to restrict the design search to preferred regions of the design space. This second class of constraint, related to design preference is not used in the decision-based collaborative optimization framework developed in this research. In this research preferred regions of the design space are imposed implicitly through the demand function.

### Collaborative Optimization (CO)

The Collaborative Optimization (CO) strategy was first proposed by Kroo, et al. (1994) and Balling and Sobieszcanski-Sobieski (1994). The CO strategy has been successfully applied to a number of different design problems including the design of a single-stage-to-orbit launch vehicle (Braun, et. al, 1996b). Tappeta and Renaud (1997) extended this approach and developed three different formulations to provide for multiobjective optimization of non-hierarchic systems.

Collaborative Optimization (CO) is a two level optimization method specifically created for large-scale distributed-analysis applications. The basic architecture of collaborative optimization (Braun, et. al, 1996a) is shown in Fig. 3. The CO framework facilitates concurrent design at the discipline design level. To achieve the concurrency in the subspace level, auxiliary design variables  $(x_{aux})_{ij}$  are introduced as additional design variables to replace the coupling variables  $y_{ij}$  so that the analyses in disciplines  $i$  and  $j$  can be executed concurrently. Compatibility constraints (or discrepancy functions)  $d$  are added to ensure consistency such that  $(x_{aux})_{ij} = y_{ij}$ . Compatibility constraints are usually in the form of equality constraints.

The system level optimizer attempts to minimize a system level objective function  $F$  while satisfying all the compatibility constraints  $d$ . System level design variables  $x_{sys}$  consist of not only the shared variables  $x_{sh}$  but also the auxiliary  $x_{aux}$  variables. These variables are specified by the system level optimizer and are sent down to subspaces as targets to be matched. Each subspace, as a local optimizer, operates on its own set of design variables  $x_{ssi}$  with the goal of matching target

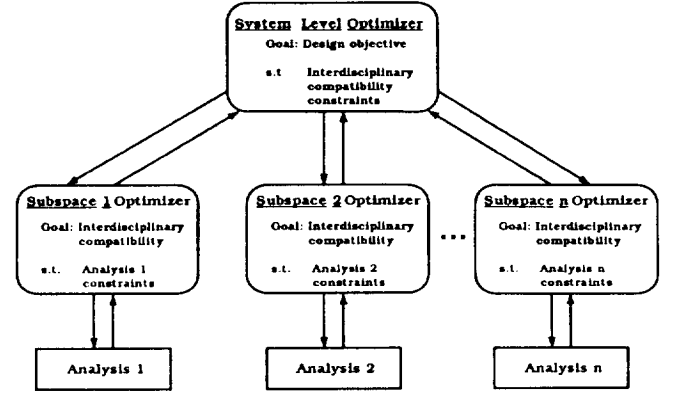


Figure 3. Basic Collaborative Optimization Architecture (Braun et. al, 1996a)

values posed by the system level as well as satisfying local constraints  $g_i$ . The matching can be attained by minimizing the discrepancy  $d_i$  between some of the local design variables and/or local states and their corresponding target values, in other words, the objective functions at subspace level are identical to the system level (compatibility) constraints. This formulation allows the use of post-optimal sensitivities calculated at the subspace optimum to be used as the gradients of the system level constraints. This important feature improves the overall efficiency of CO by eliminating the need to execute subspace analysis for the sole purpose of calculating system constraint gradients by finite differencing.

## DECISION-BASED COLLABORATIVE OPTIMIZATION

### Decision-Based CO (DBCO) Framework

In the decision-based collaborative optimization (DBCO) framework developed in this research, the multidisciplinary enterprise model (Fig. 2) is decomposed along analysis boundaries into several subsystems. The method of collaborative optimization (CO) is used to determine the optimal design of this complicated model, where an optimizer is integrated within each subsystem or discipline. The resulting decision-based collaborative optimization (DBCO) framework rigorously simulates the existing relationship between business and engineering in multidisciplinary systems design as shown in Fig. 4. A brief discussion about this framework is given below.

#### System Level Optimization

In the decision-based collaborative optimization framework, the business decisions are made at the system level. As discussed before, the goal of decision-based design optimization is to maximize the expected utility of net revenue ( $E(u(NR))$ ) of the engineering artifact being designed. The system level optimizer in the decision-based collaborative optimization framework attempts to increase expected utility of net revenue while satisfying compatibility constraints  $d$ . According to the

analyses in the business discipline and the subspace optimization results, the system level optimizer determines price  $P$  and establishes a set of performance targets for shared design variables  $x_{sh}$  and auxiliary design variables  $x_{aux}$ , including demand  $q$  and total cost  $C_T$ . Attributes  $a$  are treated as auxiliary design variables which influence demand. Note that demand  $q$  and total cost  $C_T$  are also among the auxiliary variables. Also note that since the design variable price  $P$  is only needed in the business discipline, it is not a target for any subspace.

The business discipline is operated on directly by the system level optimizer. It is *not* further decomposed into demand and utility of profit subspaces. The reasons why the business discipline is dealt with in this manner are as follows: First of all, there is no two-way coupling between the analysis in the demand subspace and the utility of profit subspace. The demand  $q$  is fed forward into the Profit subspace. Secondly, the analyses involved in this discipline are relatively simple and straightforward (Eqs. (1) and (2)). Consequently it is relatively easy to obtain sensitivity information with respect to the profit (or net revenue) or the utility of net revenue. In the cases where an analytical demand model and an analytical utility model are supplied, the sensitivity information is readily acquired through the application of the chain rule.

The system level optimization problem in its standard form is given in Eq. (3).

$$\begin{aligned}
 &\text{Minimize:} && F = -E(u(NR)) \\
 &\text{w.r.t. } x_{sys}^0 && \\
 &\text{Subject to:} && d_i^* = 0 \quad i = 1, 2, \dots, n_{ss} \\
 &&& (x_{sys}^0)_{min} \leq x_{sys}^0 \leq (x_{sys}^0)_{max} \\
 &&& x_{sys}^0 = (x_{sh}^0, x_{aux}^0) \\
 &&& P > 0
 \end{aligned} \tag{3}$$

Note that maximizing the expected utility  $E(u(NR))$  is equivalent to minimizing its negative value. Note also that net revenue (NR) is determined by the system level design variable  $x_{sys}$  and price  $P$ . Since system design variables  $x_{sys}$  are posed as targets to the subspaces, the term  $x_{sys}^0$  is used in Eq. (3) so that they can be distinguished from subspace design variables in the subproblem formulations. The term  $d_i^*$  refers to the optimal value of the discrepancy function  $d_i$  obtained by the subproblems. The formulation of  $d_i$  is discussed in the next section.

#### Subspace Level Optimization

The subspace optimizer seeks to satisfy the targets sent down by the system level optimizer and reports the discrepancy  $d_i^*$  back to the system level. Meanwhile the subspace optimizers are subject to local design constraints  $g_i$ . In the field of

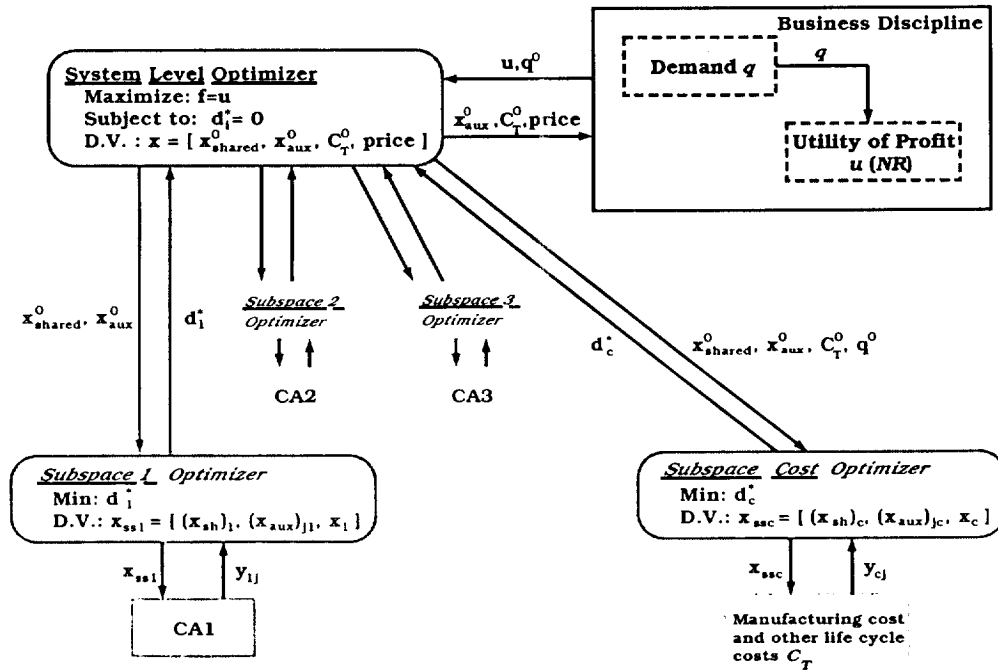


Figure 4. Decision-Based Collaborative Optimization

engineering design, the design constraints normally guard against failure or restrict the search to preferred region of the design space. One example of failure-related constraints is to require that “the axial load in a beam not exceed its buckling load”. The statement that “the mass of the beam should be less than 7 kg” is an example of constraints based on preference. The use of constraints to restrict the search to preferred region of design space is not recommended in decision-based design approach. Firstly, to impose a preferred space, the engineer must decide and quantify what level of behavior is unacceptable or undesirable (i.e., not preferable). This is a matter of decision-making and by imposing constraints of preferences, the designer is removing some degrees of freedom in the design process. The resulting system optimization may fail to identify the design with the best trade-off, especially when this constraint is active or near active at the optimal solution. In the example of the beam, a beam of mass greater than 7 kg is said to be unacceptable. But it is possible that a beam of 7.1 kg can support a much higher load than a beam of 7kg. If the goal of the optimization is to find a light beam that can support a large load, the beam of 7.1kg might be a better design. Yet if a constraint is set to ensure the mass of the beam to be no greater than 7 kg, the optimizer will not locate the beam of 7.1kg, even though it may be more preferred.

Upon closer examination, undesirable behaviors (i.e., non-preferred region) are often undesirable because such behaviors lead to a decrease in the demand of the product and/or an increase in the cost of the product, which in the end, results in a decrease of profit. In DBD the market place is used to determine preferred region of the design space through demand and cost models, and therefore constraints related to undesirability or preference are eliminated. Therefore the local constraints in the decision-based collaborative optimization (DBCO) framework tend to be those constraints that guard against system failure. Other traditional engineering constraints related to consumer preference are eliminated from the subspace optimization and are instead incorporated in the demand model and/or cost model.

The subspace optimization problem for discipline 1 of Fig. 4 in its standard form is given below:

$$\begin{aligned}
 &\text{Minimize:} \\
 &\text{w.r.t. } \mathbf{x}_{ss1} \quad d_1 = ((\mathbf{x}_{sh})_1 - (\mathbf{x}_{sh}^0)_1)^2 \\
 &\quad + \sum_{j=2}^{n_{cs}} ((\mathbf{x}_{aux})_{j1} - (\mathbf{x}_{aux}^0)_{j1})^2 \\
 &\quad + \sum_{k=2}^{n_{cs}} (y_{1k} - (\mathbf{x}_{aux}^0)_{1k})^2 \\
 &\text{Subject to:} \quad g_1 \geq 0 \\
 &\quad (\mathbf{x}_{ss1})_{min} \leq \mathbf{x}_{ss1} \leq (\mathbf{x}_{ss1})_{max} \\
 &\quad \mathbf{x}_{ss1} = ((\mathbf{x}_{sh})_1, (\mathbf{x}_{aux})_{j1}, \mathbf{x}_1)
 \end{aligned} \tag{4}$$

The subspace must satisfy local constraints  $g_i$  while attempting to minimize discrepancies in system level targets. Note that attribute targets are imposed in the second and the third terms of the discrepancy function.

### System Level Constraints Gradient

The gradient of system level constraints plays an important role in forming search directions for the system level optimization. As mentioned earlier, one important feature of CO is that post-optimality sensitivity analysis from the converged subspace optimization problem can be used to provide system level derivatives for compatibility constraints (Kroo et. al, 1994). As a result, both computational expense and numerical error are reduced. This is possible because the system level design variables are treated as parameters (i.e., targets) in the subproblems. Note that for a certain discipline  $i$ , depending on the contributing analysis involved, not all the system level design variables  $\mathbf{x}_{sys}$  are necessarily posed as targets to be matched. It is possible that only a subset of  $\mathbf{x}_{sys}$ , referred to as  $(\mathbf{x}_{sys})_i$ , is sent down as subspace  $i$  targets. Generally all the subsets for  $n_{ss}$  subspaces are not mutually exclusive, i.e., their intersections exist. The set of system level design variables  $\mathbf{x}_{sys}$  is the union of all the subsets  $(\mathbf{x}_{sys})_i$ . The gradient of system level constraint  $d_i^*$  with respect to the subset  $(\mathbf{x}_{sys})_i$  of the system level design variables sent down as targets to discipline  $i$  is given below in Eq. (5). The gradient of system level constraint  $d_i^*$  with respect to those system level design variables which are not imposed as targets for discipline  $i$  is apparently zero.

$$\frac{\partial d_i^*}{\partial (\mathbf{x}_{sys})_i} = -2[(\mathbf{x}_{ss}^*)_i - (\mathbf{x}_{sys}^0)_i] \tag{5}$$

The term  $(\mathbf{x}_{ss}^*)_i$  refers to the vector formed by the converged optimal values of local variables and states in discipline  $i$  at the end of subspace optimization. The elements in  $(\mathbf{x}_{ss}^*)_i$  are the optimal counterparts of the system level targets  $(\mathbf{x}_{sys})_i$ .

### Test Problem

A preliminary application of the decision-based collaborative optimization framework has been tested on an Aircraft Concept Sizing (ACS) problem. This problem was originally developed by the MDO research group at the University of Notre Dame (Wujek, Renaud, Batill, et. al., 1996). It involves the preliminary sizing of a general aviation aircraft subject to certain performance constraints. The design variables in this problem are comprised of variables relating to the geometry of the aircraft, propulsion and aerodynamic characteristics, and flight regime. Appropriate bounds are placed on all design variables.



The problem also includes a number of parameters which are fixed during the design process to represent constraints on mission requirements, available technologies, and aircraft class regulations.

The original problem has ten design variables and five parameters. The design of the system is decomposed into six contributing analyses. This problem has been modified by Tappeta (1996) to fit the framework of multiobjective coupled MDO systems. It is further modified in this research to be suitable for the case of decision-based design (DBD). For comparison, the rest of this section gives a brief description of the modified ACS problem by Tappeta. It will be referred to as the ACS problem from hereon. The DBD version of the ACS problem will be discussed in the following sections.

The Aircraft Concept Sizing (ACS) problem has three disciplines (see Fig. 5): aerodynamics (CA1), weight (CA2) and performance (CA3). It can be observed from the dependency diagram that the system has two feed-forwards and there are no feed-backs between disciplines. Table 1 lists the design variables and their bounds in the ACS problem. Table 2 lists the usage of design variables as inputs to each discipline. It can be seen that there are five shared design variables ( $x_1 \sim x_4$  and  $x_7$ ). Table 3 lists the parameters and their values. Table 4 lists the state variables and their relations with each discipline. Clearly there are two coupled states ( $y_2$  and  $y_4$ ). Table 5 contains all the relevant information for the ACS problem in the standard MDO standard notation introduced earlier.

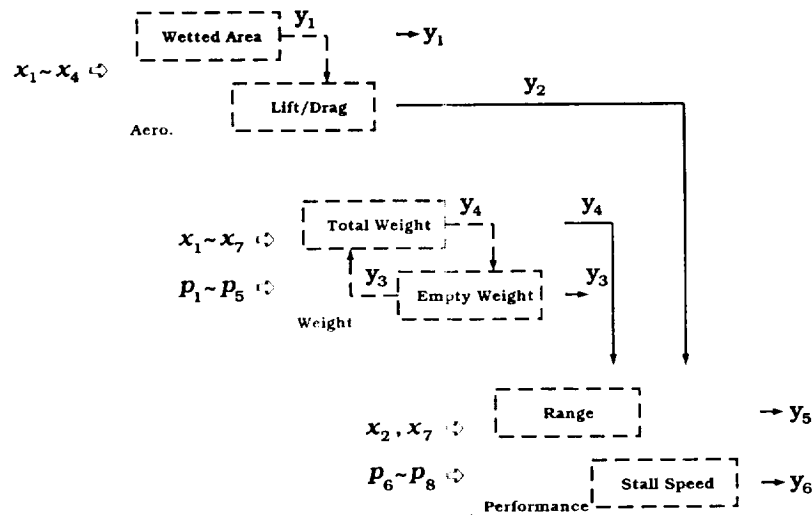
**Table 1: List of Design Variables in ACS Problem**

	Name (Unit)	L	U
$x_1$	aspect ratio of the wing	5	9
$x_2$	wing area (ft <sup>2</sup> )	100	300
$x_3$	fuselage length (ft)	20	30
$x_4$	fuselage diameter (ft)	4	5
$x_5$	density of air at cruise altitude (slugs/ft <sup>3</sup> )	.0017	.002378
$x_6$	cruise speed (ft/sec)	200	300
$x_7$	fuel weight (lbs)	100	2000

**Table 2: Input Design Variables of Each Discipline in ACS\***

	CA1 (Aero.)	CA2 (Weight)	CA3 (Perf.)
$x_1$	√	√	
$x_2$	√	√	√
$x_3$	√	√	
$x_4$	√	√	
$x_5$		√	
$x_6$		√	
$x_7$		√	√

\*Shaded cells in the table indicate shared variables.



**Figure 5. Aircraft Concept Sizing Problem**

Table 3: List of Parameters in ACS Problem

	Name	Description	Value
$p_1$	$N_{pass}$	number of passengers	2
$p_2$	$N_{en}$	number of engines	1
$p_3$	$W_{en}$	engine weight	197 (lbs)
$p_4$	$W_{pay}$	payload weight	398 (lbs)
$p_5$	$N_{zult}$	ultimate load factor	5.7
$p_6$	Eta	propeller efficiency	.85
$p_7$	c	specific fuel consumption	.4495 (lbs/hr/hp)
$p_8$	$C_{l_{max}}$	maximum lift coeff. of the wing	1.7

Table 4: Lists of States in ACS Problem\*

	Name (Unit)	Output From	Input To
$y_1$	total aircraft wetted area (ft <sup>2</sup> )	CA1	
$y_2$	max lift to drag ratio	CA1	CA3
$y_3$	empty weight (lbs)	CA2	
$y_4$	gross take-off weight (lbs)	CA2	CA3
$y_5$	aircraft range (miles)	CA3	
$y_6$	stall speed (ft/sec)	CA3	

\*Shaded cells in the table indicate coupling states.

The objective in the ACS problem is to determine the least gross take-off weight within the bounded design space subject to two performance constraints. The first constraint is that the aircraft range must be no less than a prescribed requirement, and the second constraint is that the stall speed must be no greater than a specified maximum stall speed. In standard form, the optimization problem is given below:

$$\begin{aligned}
 &\text{Minimize:} && F = \text{Weight} = y_4 \\
 &\text{Subject to:} && g_1 = 1 - \frac{y_6}{V_{stall_{req}}} \geq 0 \\
 &&& g_2 = 1 - \frac{\text{Range}_{req}}{y_5} \geq 0 \\
 &&& V_{stall_{req}} = 70 \text{ ft/sec} \\
 &&& \text{Range}_{req} = 560 \text{ miles}
 \end{aligned} \tag{6}$$

Table 5: Design Vectors and Functions for ACS Problem

	Vector or Function	Variables or Content
	$x$	$[x_1, x_2, x_3, x_4, x_5, x_6, x_7]$
	$x_{sh}$	$[x_1, x_2, x_3, x_4, x_7]$
	$x_{aux}$	goals for $y_2, y_4$
	$x_{sys}^o$	$[x_1^o, x_2^o, x_3^o, x_4^o, x_7^o, y_2^o, y_4^o]$
	$F$	$F=y_4$
CA1	system targets to be matched	$(x_{sys}^o)_1 = [x_1^o, x_2^o, x_3^o, x_4^o, y_2^o]$
	$x_1$	empty set
	$(x_{sh})_1$	$[x_1, x_2, x_3, x_4]$
	$(x_{aux})_1$	empty set
	$x_{ss1}$	$[x_1, x_2, x_3, x_4]$
	$g_1$	empty set
	analysis	$[y_1, y_2] = CA1[x_1, x_2, x_3, x_4]$
CA2	optimal target values	$(x_{ss}^*)_1 = [x_1^*, x_2^*, x_3^*, x_4^*, y_2^*]$
	system targets to be matched	$(x_{sys}^o)_2 = [x_1^o, x_2^o, x_3^o, x_4^o, x_7^o, y_4^o]$
	$x_2$	$[x_5, x_6]$
	$(x_{sh})_2$	$[x_1, x_2, x_3, x_4, x_7]$
	$(x_{aux})_2$	empty set
	$x_{ss2}$	$[x_1, x_2, x_3, x_4, x_5, x_6, x_7]$
	$g_2$	empty set
CA3	analysis	$[y_3, y_4] = CA2[x_1, x_2, x_3, x_4, x_5, x_6, x_7]$
	optimal target values	$(x_{ss}^*)_2 = [x_1^*, x_2^*, x_3^*, x_4^*, x_7^*, y_4^*]$
	system targets to be matched	$(x_{sys}^o)_3 = [x_2^o, x_7^o, y_2^o, y_4^o]$
	$x_3$	empty set
	$(x_{sh})_3$	$[x_2, x_7]$
	$(x_{aux})_3$	$[y_2, y_4]$
	$x_{ss3}$	$[x_2, x_7, y_2, y_4]$
CA3	$g_3$	$[g_1, g_2]$
	analysis	$[y_5, y_6] = CA3[x_2, x_7, y_2, y_4]$
	optimal target values	$(x_{ss}^*)_3 = [x_2^*, x_7^*, y_2^*, y_4^*]$

## Demand Model and Cost Model

The approach of decision-based design requires engineers to not only focus on the product performance but also life-cycle costs as well as demand and the profit obtained over the life-cycle of the product. Thus it is very important to construct a proper demand model and a proper cost model for the product. The authors are aware that the task to build such models is not an easy one, and engineers are generally not trained for this task. Since this research is concentrated on the optimization aspect of decision-based design, it is reasonable to assume that other discipline experts have developed such demand and cost models and made them available to the optimizer.

In the case of the Aircraft Concept Sizing (ACS) problem, neither a demand model nor a cost model was available from the previous studies. In order to apply the decision-based collaborative optimization framework of Fig. 4, a demand model and a cost model have been developed. These models are built in a way such that they agree with industry trends for this specific class of aircraft and that they lead to reasonable optimization behavior. Although they are by no means complete, they serve fairly well as concept models for the application of DBCO at the current stage. Figure 6 illustrates the demand and cost models of the ACS problem. Only the annual demand, annual cost and annual profit are considered in the current research.

## Demand Model

The first step in building the demand model is to identify the attributes that influence the demand  $q$  of this aircraft. The conventional optimization ACS problem (Eq. (6)) tries to minimize gross take-off weight ( $y_4$ ) while satisfying two performance constraints, one on stall speed ( $y_6$ ) and the other on aircraft range ( $y_5$ ). Closer examination reveals that the objective function and two constraints, imposed in the original problem, are based on the designer's estimate of customer preference for weight, stall speed and range. In the decision-based design approach, it is more appropriate to treat these quantities (take-off weight  $y_4$ , aircraft range  $y_5$ , and stall speed  $y_6$ ) as attributes of demand. Hence there are no performance constraints in the DBD version of the ACS problem, and the goal of the optimization is to maximize profit.

It is also assumed that customers are interested in the cruise speed ( $x_6$ ) of the aircraft as well as how much room they would have on the airplane. A new state variable, fuselage volume ( $y_7$ ), is introduced to reflect the concern for passenger room on the aircraft. In all, there are five attributes of demand in the DBD version of ACS problem: take-off weight  $y_4$ , aircraft range  $y_5$ , stall speed  $y_6$ , fuselage volume  $y_7$ , and cruise speed  $x_6$ . Note that demand is also influenced by price  $P$ .

The demand model developed is a multiplicative model:

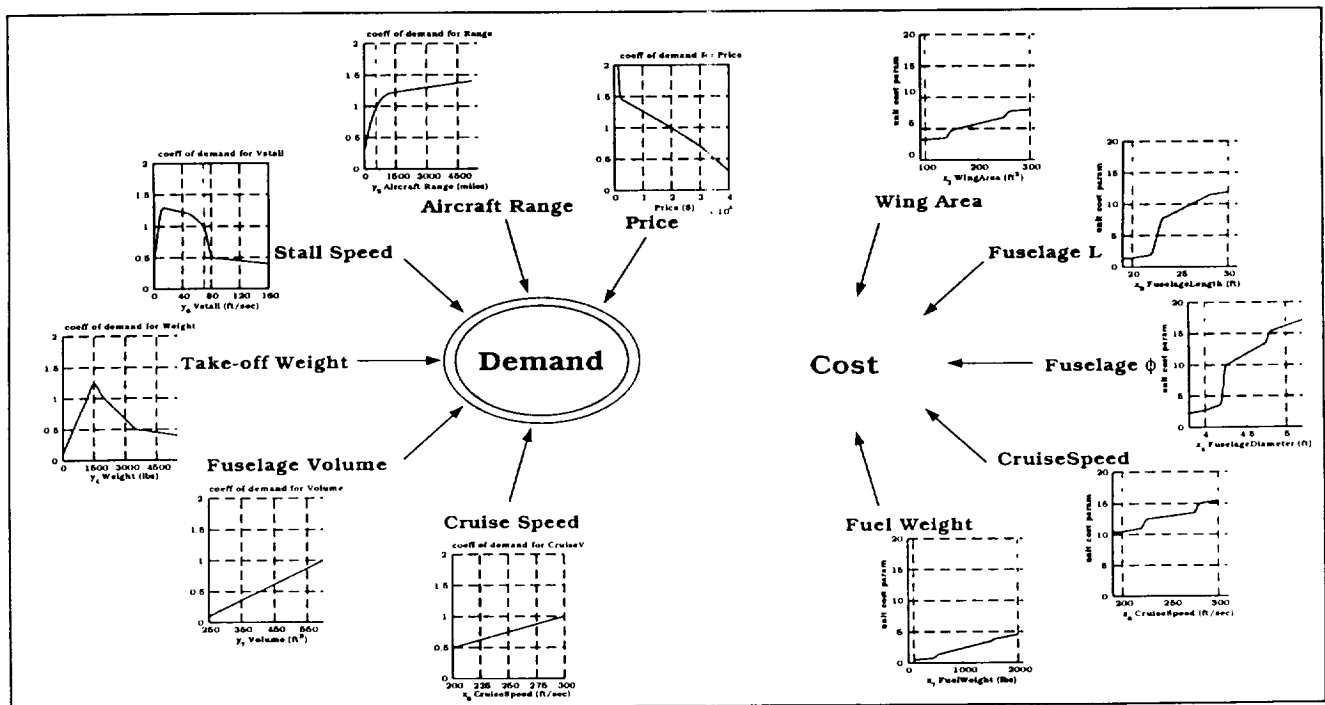


Figure 6. Concept Demand Model and Concept Model

$$q = \bar{q}(a, P) = q_B \left( \prod_{i=1}^{n_a} c_i \right) c_{price} \quad (7)$$

where  $a = \{y_4, y_5, y_6, y_7, x_6\}$   
 $n_a = 5$   
 $q_B = 1200$

The term  $q_B$  represents a baseline demand, which is set to 1200. The number of attributes is denoted by  $n_a$ . The affect of each attribute on the final demand is reflected by the demand coefficient  $c_i$ . Similarly the term  $c_{price}$  denotes the demand coefficient of price  $P$ . The final demand  $q$  is the product of all demand coefficients and the baseline demand  $q_B$ .

Demand coefficients for each attribute and price are developed by financial analysts and marketing personnel within the business discipline and vary with time. For the purpose of this study, they are assumed fixed with respect to time and are given in Fig. 6. The curves in Fig. 6 plot the coefficient of demand on the ordinate and the corresponding attribute (or price) on the abscissa.

1. gross take-off weight ( $y_4$ )  
The lower the take-off weight, the higher the demand; but an aircraft with a very light weight is not desired.
2. aircraft range ( $y_5$ )  
The longer the aircraft range, the higher the demand; but after the range reaches more than 600 miles, there is no significant increase in demand when range increases. This formulation is not unlike the original performance constraint  $g_2$  (Eq. (6)), where the coefficient is set to 1 when aircraft range equals to 560 miles.
3. stall speed ( $y_6$ )  
The lower the stall speed, the higher the demand; but a near-zero stall speed is not necessary. This formulation is not unlike the original performance constraint  $g_1$  (Eq. (6)), where the coefficient is set to 1 when stall speed equals to 70 ft/sec.
4. fuselage volume ( $y_7$ )  
The larger the fuselage volume, the higher the demand.
5. cruise speed ( $x_6$ )  
The faster the cruise speed, the higher the demand.
6. price ( $P$ )  
The lower the price, the higher the demand; if the aircraft is sold for free ( $P=0$ ), the demand approaches infinity.

#### Cost Model

It is assumed that all costs ( $C_T$ ) related to the production of the aircraft can be divided into two categories: fixed cost  $C_F$  and variable cost  $C_V$  (Krajewski and Ritzman, 1999). The fixed cost

$C_F$  is the part of the total cost  $C_T$  that remains consistent regardless of changes in the amount of product produced, for example, the annual cost of renting or buying equipment or facilities. The variable cost  $C_V$  is the portion of the total cost  $C_T$  that varies directly with quantity of product produced, such as cost per unit for material and labor. If we assume the quantity of product produced and sold per year is equal to the demand  $q$  for the product per year, the total cost  $C_T$  per year is:

$$C_T = C_F + q C_V \quad (8)$$

It is assumed that the variable cost  $C_V$  in the ACS problem is dependent on five of the seven design variables including wing area ( $x_2$ ), fuselage length ( $x_3$ ), fuselage diameter ( $x_4$ ), cruise speed ( $x_6$ ) and fuel weight ( $x_7$ ). A variable cost parameter  $(p_V)_i$  is assigned to each cost-related variable to represent the portion of variable cost (per unit) associated with each variable. The total variable cost per unit is the sum of all variable cost parameters:

$$C_V = \sum_{i=1}^{n_{cost}} (p_V)_i \quad (9)$$

where  $n_{cost}$  is the number of cost-related variables. The guideline for assigning variable cost parameters is: the larger the variable, the higher the cost. The curves in Fig. 6, associated with each cost-related variable, plot the variable cost parameter (unit: 10,000 dollars) on the ordinate and the corresponding cost-related variable on the abscissa. The step jumps in the curves represents the need to purchase (or rent) and/or install new equipment (or facilities) when the size of the aircraft exceeds existing production capabilities.

Substituting Eq. (9) into (8), the model of the total cost  $C_T$  is:

$$C_T = C_F + q \sum_{i=1}^{n_{cost}} (p_V)_i \quad (10)$$

and the unit cost  $C_u$  can be obtained by dividing both sides of Eq. (10) by demand  $q$ :

$$C_u = \frac{C_F}{q} + \sum_{i=1}^{n_{cost}} (p_V)_i \quad (11)$$

Note that the number of product produced ( $q$ ) may have a discounting effect on the variable cost parameters  $p_i$ . For instance, usually the cost per unit for material will decrease when the total amount of material bought increases. Thus a  $q$ -discounting option has been included in the determination of variable cost parameter in the cost model.

### Note

The demand and cost models developed in this paper are by no means complete. They are conceptual and rather simplistic. Future work on the modification of these models will include (and not limit to) the following issues:

#### 1. gross take-off weight ( $y_4$ )

It has been pointed out to the authors that to a customer higher gross weight is actually desirable because it leads to longer aircraft range. Meanwhile higher gross weight leads to higher manufacture cost. Therefore a modified demand model would include the gross take-off weight as a slightly favorable feature (i.e., the higher the take-off weight, the higher the demand). On the other hand a modified cost model may include the gross take-off weight as a strongly negative factor (i.e., the higher the take-off weight, the higher the cost).

#### 2. aspect ratio of the wing ( $x_1$ )

Increasing aspect ratio will increase the wing structural weight, which will in turn lead to an increase in the aircraft gross take-off weight, thus add to the total cost. A modified cost model would include aspect ratio as another negative factor.

#### 3. price ( $P$ )

It has been brought to the authors' attention that in the real world, due to the maintenance requirements such as insurance and hanger, the demand will not approach infinity if the aircraft is given out for free. A modified demand model will address this issue by assigning a definite number to the demand coefficient for price when price is set to zero. This definite number will be associated with the maximum demand possible for this aircraft.

### DBCO Formulation

The decision-based collaborative optimization framework has been applied to the DBD version of the Aircraft Concept Sizing (ACS) problem. This application is a preliminary study, and focuses on the collaborative optimization feature of the DBCO framework. The issues of propagated uncertainty are neglected in this study. The utility of profit is assumed to be the profit itself. Hence the objective of the resulting deterministic optimization is to maximize profit (or net revenue). During the optimization, the demand  $q$  is treated as a continuous variable, rather than an integer. At the end of the system optimization,  $q$  is rounded to the nearest integer.

Two additional disciplines are added in the DBD version of the ACS problem: cost (CAc) and business (CAb). Price  $P$  is a new design variable and a new state variable (fuselage volume  $y_7$ ) is introduced. Table 6 provides the list of input design variables to each discipline in the DBD version of ACS problem. Clearly design variable  $x_6$  (cruise speed) enters the set of shared variables. Table 7 lists the states  $y$ , demand  $q$ , total

cost  $C_T$  and net revenue  $NR$ . It also depicts how they are related to each discipline. The set of coupling variables expands to include five additional members:  $y_5$  (aircraft range),  $y_6$  (stall speed),  $y_7$  (fuselage volume),  $q$  (demand) and  $C_T$  (total cost). Table 8 contains all the design vector information for the DBD version of subspaces 1 (aerodynamics), 2 (weight), 3 (performance) and c (cost) of the ACS problem in MDO standard notation. Table 9 contains the design vector information of subspace b (business) since it is operated directly by the system level optimizer. The difference between the DBD version of ACS problem and the modified ACS problem by Tappeta (1996) can be clearly observed by comparing Tables 6, 7, 8 and 9 with Tables 2, 4 and 5, respectively.

**Table 6: Input Design Variables to Each Discipline in ACS (DBD version)\***

	CA1 (Aero.)	CA2 (Weight)	CA3 (Perf.)	CAc (Cost)	CAb (Busin.)
$x_1$	√	√			
$x_2$	√	√	√	√	
$x_3$	√	√		√	
$x_4$	√	√		√	
$x_5$		√			
$x_6$		√		√	√
$x_7$		√	√	√	
$P$					√

\*Shaded cells indicate shared variables.

**Table 7: Lists of States in ACS Problem (DBD version)\***

	Output From	Input To
$y_1$	CA1	
$y_2$	CA1	CA3
$y_3$	CA2	
$y_4$	CA2	CA3, CAb
$y_5$	CA3	CAb
$y_6$	CA3	CAb
$y_7$	CA1	CAb
$q$	CAb	CAc
$C_T$	CAc	CAb
$NR$	CAb	

\*Shaded calls indicate coupling states.

**Table 8: Design Vectors for ACS Problem (DBD version)**

	Vector or Function	Variables or Content
	$x$	$[x_1, x_2, x_3, x_4, x_5, x_6, x_7, P]$
	$x_{sh}$	$[x_1, x_2, x_3, x_4, x_6, x_7]$
	$x_{aux}$	goals for $y_2, y_4, y_5, y_6, y_7, C_T, q$
	$x_{sys}^0$	$[x_1^0, x_2^0, x_3^0, x_4^0, x_6^0, x_7^0, y_2^0, y_4^0, y_5^0, y_6^0, y_7^0, C_T^0, q^0]$
	$F$	$F = -NR$
CA1	system targets to be matched	$(x_{sys}^0)_1 = [x_1^0, x_2^0, x_3^0, x_4^0, y_2^0, y_7^0]$
	$x_1$	empty set
	$(x_{sh})_1$	$[x_1, x_2, x_3, x_4]$
	$(x_{aux})_1$	empty set
	$x_{ss1}$	$[x_1, x_2, x_3, x_4]$
	$g_1$	empty set
	analysis	$[y_1, y_2, y_7] = CA1[x_1, x_2, x_3, x_4]$
CA2	optimal target values	$(x_{ss}^*)_1 = [x_1^*, x_2^*, x_3^*, x_4^*, y_2^*, y_7^*]$
	system targets to be matched	$(x_{sys}^0)_2 = [x_1^0, x_2^0, x_3^0, x_4^0, x_6^0, x_7^0, y_4^0]$
	$x_2$	$[x_5]$
	$(x_{sh})_2$	$[x_1, x_2, x_3, x_4, x_6, x_7]$
	$(x_{aux})_2$	empty set
	$x_{ss2}$	$[x_1, x_2, x_3, x_4, x_5, x_6, x_7]$
	$g_2$	empty set
CA3	analysis	$[y_3, y_4] = CA2[x_1, x_2, x_3, x_4, x_5, x_6, x_7]$
	optimal target values	$(x_{ss}^*)_2 = [x_1^*, x_2^*, x_3^*, x_4^*, x_6^*, x_7^*, y_4^*]$
	system targets to be matched	$(x_{sys}^0)_3 = [x_2^0, x_7^0, y_2^0, y_4^0, y_5^0, y_6^0]$
	$x_3$	empty set
	$(x_{sh})_3$	$[x_2, x_7]$
	$(x_{aux})_3$	$[y_2, y_4]$
	$x_{ss3}$	$[x_2, x_7, y_2, y_4]$
CA3	$g_3$	empty set
	analysis	$[y_5, y_6] = CA3[x_2, x_7, y_2, y_4]$
	optimal target values	$(x_{ss}^*)_3 = [x_2^*, x_7^*, y_2^*, y_4^*, y_5^*, y_6^*]$

**Table 8: Design Vectors for ACS Problem (DBD version) (Continued)**

CAc	system targets to be matched	$(x_{sys}^0)_c = [x_2^0, x_3^0, x_4^0, x_6^0, x_7^0, q^0, C_T^0]$
	$x_c$	empty set
	$(x_{sh})_c$	$[x_2, x_3, x_4, x_6, x_7]$
	$(x_{aux})_c$	$[q]$
	$x_{ssc}$	$[x_2, x_3, x_4, x_6, x_7, q]$
	$g_c$	empty set
	analysis	$C_T = CAc[x_2, x_3, x_4, x_6, x_7, q]$
	optimal target values	$(x_{ss}^*)_c = [x_2^*, x_3^*, x_4^*, x_6^*, x_7^*, q^*, C_T^*]$

**Table 9: Design Vectors in the subspace b (business) for ACS Problem (DBD version)**

CAB	targets sent down to other subspaces	$(x_{sys}^0)_b = [x_6^0, y_4^0, y_5^0, y_6^0, y_7^0, q^0, C_T^0]$
	$x_b$	$[P]$
	$(x_{sh})_b$	$[x_6]$
	$(x_{aux})_b$	$[y_4, y_5, y_6, y_7, C_T]$
	$x_{ssb}$	$[x_6^0, y_4^0, y_5^0, y_6^0, y_7^0, C_T^0, P]$
	$g_b$	empty set
	analysis	$[q^0, NR] = CAB[x_6^0, y_4^0, y_5^0, y_6^0, y_7^0, C_T^0, P]$

The system level optimization problem, for this application, in its standard form is detailed in Eq. (12).

#### System Level Optimization

Minimize:  $F = -NR$   
w.r.t.  $x_{sys}^0$

Subject to:  $d_1^* = 0$

$d_2^* = 0$

$d_3^* = 0$

$d_c^* = 0$

$(x_{sys}^0)_{min} \leq x_{sys}^0 \leq (x_{sys}^0)_{max}$

where  $x_{sys}^0 = [x_1^0, x_2^0, x_3^0, x_4^0, x_6^0, x_7^0,$

$y_2^0, y_4^0, y_5^0, y_6^0, y_7^0, C_T^0, P]$

$q^0 = \bar{q}(y_4^0, y_5^0, y_6^0, y_7^0, x_6^0, P)$

$NR = P \cdot q^0 - C_T^0$

(12)

Note that the system level optimizer calls the business discipline directly to obtain demand  $q^0$  and the system level objective  $NR$ . There are thirteen system level design variables and four compatibility constraints that are evaluated in subspace 1, 2, 3 and c.

The subspace optimization problems for each discipline in their standard forms are given by Eqs. (13) ~ (16).

#### Subspace 1 (Aerodynamics) Optimization

$$\begin{aligned}
 &\text{Minimize:} \\
 &\text{w.r.t. } \mathbf{x}_{ss1} \quad d_1 = (x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 \\
 &\quad + (x_3 - x_3^0)^2 + (x_4 - x_4^0)^2 \\
 &\quad + (y_2 - y_2^0)^2 + (y_7 - y_7^0)^2 \\
 &\text{Subject to:} \quad (\mathbf{x}_{ss1})_{min} \leq \mathbf{x}_{ss1} \leq (\mathbf{x}_{ss1})_{max} \\
 &\quad \text{where} \quad \mathbf{x}_{ss1} = [x_1, x_2, x_3, x_4] \\
 &\quad [y_1, y_2, y_7] = CA1[x_1, x_2, x_3, x_4]
 \end{aligned} \tag{13}$$

#### Subspace 2 (Weight) Optimization

$$\begin{aligned}
 &\text{Minimize:} \\
 &\text{w.r.t. } \mathbf{x}_{ss2} \quad d_2 = (x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 \\
 &\quad + (x_3 - x_3^0)^2 + (x_4 - x_4^0)^2 \\
 &\quad + (x_6 - x_6^0)^2 + (x_7 - x_7^0)^2 \\
 &\quad + (y_4 - y_4^0)^2 \\
 &\text{Subject to:} \quad (\mathbf{x}_{ss2})_{min} \leq \mathbf{x}_{ss2} \leq (\mathbf{x}_{ss2})_{max} \\
 &\quad \text{where} \quad \mathbf{x}_{ss2} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7] \\
 &\quad [y_3, y_4] = CA2[x_1, x_2, x_3, x_4, x_5, x_6, x_7]
 \end{aligned} \tag{14}$$

#### Subspace 3 (Performance) Optimization

$$\begin{aligned}
 &\text{Minimize:} \\
 &\text{w.r.t. } \mathbf{x}_{ss3} \quad d_3 = (x_2 - x_2^0)^2 + (x_7 - x_7^0)^2 \\
 &\quad + (y_2 - y_2^0)^2 + (y_4 - y_4^0)^2 \\
 &\quad + (y_5 - y_5^0)^2 + (y_6 - y_6^0)^2 \\
 &\text{Subject to:} \quad (\mathbf{x}_{ss3})_{min} \leq \mathbf{x}_{ss3} \leq (\mathbf{x}_{ss3})_{max} \\
 &\quad \text{where} \quad \mathbf{x}_{ss3} = [x_2, x_7, y_2, y_4] \\
 &\quad [y_5, y_6] = CA3[x_2, x_7, y_2, y_4]
 \end{aligned} \tag{15}$$

#### Subspace c (Cost) Optimization

$$\begin{aligned}
 &\text{Minimize:} \\
 &\text{w.r.t. } \mathbf{x}_{ssc} \quad d_c = (x_2 - x_2^0)^2 + (x_3 - x_3^0)^2 \\
 &\quad + (x_4 - x_4^0)^2 + (x_6 - x_6^0)^2 \\
 &\quad + (x_7 - x_7^0)^2 \\
 &\quad + (q - q^0)^2 + (C_T - C_T^0)^2 \\
 &\text{Subject to:} \quad (\mathbf{x}_{ssc})_{min} \leq \mathbf{x}_{ssc} \leq (\mathbf{x}_{ssc})_{max} \\
 &\quad \text{where} \quad \mathbf{x}_{ssc} = [x_2, x_3, x_4, x_6, x_7, q] \\
 &\quad C_T = CAc[x_2, x_3, x_4, x_6, x_7, q]
 \end{aligned} \tag{16}$$

Note that other than variable bounds, there are no local constraints for the subspace optimization problems.

### Optimization Results & Discussion

A Sequential Quadratic Programming (SQP) method was used for optimization in both the system level and the subspace optimization. The SQP solver, *fmincon*, was obtained from the Matlab Optimization Toolbox. The program converged to an optimum after thirty-seven system level iterations. The optimal solution is listed in Table 10. Figure 7 shows the system level optimization history of convergence of the system level objective function (negative of profit, in subplot 8), the convergence history of the four compatibility (discrepancy) constraints ( $d_1^*$ ,  $d_2^*$ ,  $d_3^*$ , and  $d_c^*$ , in subplots 9~12), and the convergence history of the seven system level design variables (cruise speed  $x_6^0$ , aircraft range  $y_5^0$ , stall speed  $y_6^0$ , fuselage volume  $y_7^0$ , price  $P$ , demand  $q^0$  and total cost  $C_T^0$ , in subplots 1~7). The abscissa of each subplot is the number of system level iterations. Note that the value of profit (not the negative of profit) was plotted in subplot 8 for easy reading. For the same reason unit cost  $C_u^0$  was plotted instead of total cost  $C_T^0$  in subplot 7.

As can be seen from the convergence plots, the system level optimizer tries to minimize both the negative of profit and the constraint violations simultaneously. At the beginning of the optimization, the system level optimizer sets targets high for price, high for the levels of performance (to ensure high demand) and low for cost based on the results of the business analyses. However these targets conflict with one another and lead to a large discrepancy at the subspace level. Thus the system level optimizer, while trying to keep profit as high as possible, was forced to lower price, downgrade performance and tolerate higher cost so that the subspace discrepancy could be reduced. Gradually the system level optimizer found the best trade-off among the targets and reached a consistent optimal design. The optimization history observed in the ACS problem

resembles the existing relationship between business and engineering in multidisciplinary systems design.

The demand model and the cost model play an important role in the decision-based design approach. In order to illustrate the influence of the demand and cost models, a conventional all-at-once optimization was performed according to the problem formulation in Eq. (6). The conventional optimum obtained is also listed in Table 10. Note that fuselage volume ( $y_7$ ) at the conventional optimum is determined by the optimal fuselage length ( $x_3$ ) and optimal fuselage diameter ( $x_4$ ). It can be observed that the conventional optimum outperforms the DBCO optimal design on lower weight ( $y_3$ ,  $y_4$ ). However it possess poor characteristics in many aspects such as smaller aircraft range ( $y_5$ ), higher stall speed ( $y_6$ ) and smaller fuselage volume ( $y_7$ ). Such an outcome is no surprise since the main concern of the conventional ACS problem is to minimize take-off weight, while the DBD approach takes into account other performance attributes, because of the DBD objective of maximizing profit.

If we assume that the aircraft configuration at the conventional optimum design will be sold at the same price as the DBD optimum design, the demand, cost and profit of the conventional product can be obtained according to the demand model (Eq. (7)) and cost model (Eqs. (10) and (11)) developed earlier. These values are listed in Table 10 in parentheses because of the assumption. Notice that the unit cost of conventional optimal design is lower than the unit cost of the DBD optimal design. However the poor performance attributes cause the demand for conventional optimal design to be much lower than the DBD optimal design. Hence the DBD optimal design leads to higher profit.

Table 10: Optimal Solutions for ACS Problem

	Name (Unit)	DV Bounds	DBCO Optimum	Conven. Optimum
$x_1$	aspect ratio of the wing	5~9	7.968	5
$x_2$	wing area (ft <sup>2</sup> )	100~300	230.3	176.53
$x_3$	fuselage length (ft)	20~30	21.927	20
$x_4$	fuselage diameter (ft)	4~5	4.1871	4
$x_5$	density of air at cruise altitude (slugs/ft <sup>3</sup> )	.0017~.002378	.0023	.0017
$x_6$	cruise speed (ft/sec)	200~300	219.65	200
$x_7$	fuel weight (lbs)	100~2000	231.22	142.86
$y_1$	total aircraft wetted area (ft <sup>2</sup> )		887.21	710.3
$y_2$	max lift to drag ratio		14.273	10.971
$y_3$	empty weight (lbs)		1556.6	1207.6
$y_4$	gross take-off weight (lbs)		2185.9	1748.4
$y_5$	aircraft range (miles)		953.67	560
$y_6$	stall speed (ft/sec)		68.525	70
$y_7$	fuselage volume (ft <sup>3</sup> )		301.92	251.33
$P$	price (\$)		3.56e5	(3.56e5)
$q$	demand		87	(32)
$C_T$	total cost (\$)		2.02e7	(6.39e6)
$C_u$	unit cost (\$)		2.31e5	(2.02e5)
$NR$	net revenue or profit (\$)		10.9e6	(6.39e6)

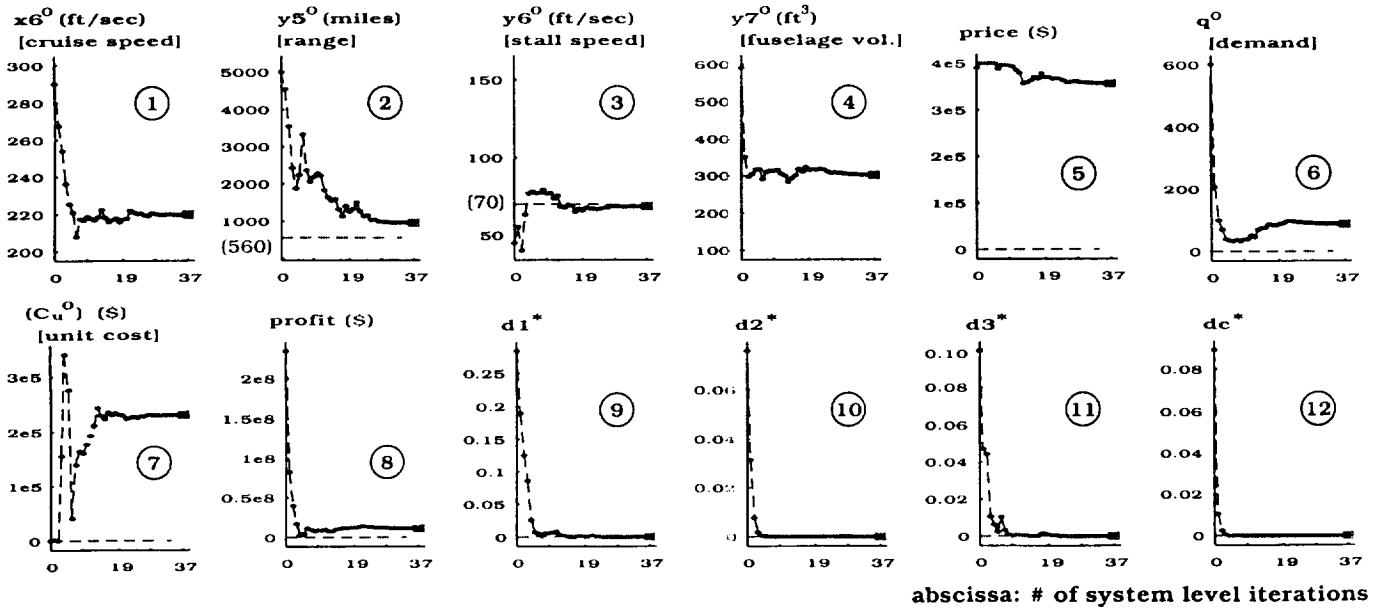


Figure 7. System Level Convergence Plots for ACS Problem (DBD version)



## CONCLUSIONS

In this research a Decision-Based Collaborative Optimization (DBCO) framework which incorporates the concepts of normative decision-based design (DBD) and the strategies of Collaborative Optimization (CO) has been developed. This bi-level non-deterministic optimization framework more accurately captures the existing relationship between business and engineering in multidisciplinary systems design. The business decisions are made at the system level, which result in a set of engineering performance targets that disciplinary engineering design teams seek to satisfy as part of subspace optimizations. The objective of the Decision-Based Collaborative Optimization (DBCO) is to maximize the expected von Neuman-Morgenstern (vN-M) utility of the profit or net revenue (NR) of a product.

A preliminary application of this approach (deterministic case) has been conducted on a multidisciplinary test problem named the Aircraft Concept Sizing (ACS) test problem. Conceptual demand and cost models have been developed. The corresponding optimization results are discussed and compared with the conventional optimization solutions.

Future work is being targeted towards a non-deterministic implementation of the DBCO framework in which the issues of propagated uncertainty in such a bi-level optimization framework will be addressed (Gu and Renaud, 2001). A conceptual utility model for the net revenue (NR) will be adapted from the literature in the field of decision-based design (DBD). The conceptual demand and cost models developed in this paper will be modified to better reflect the real world.

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## APPENDIX I: GRANT ACTIVITIES

While the focus of this grant effort was the development of a Decision-Based Collaborative Optimization (DBCO) framework for decision-based design (DBD), numerous other research efforts were supported in part by this grant. The following section details publications resulting from this grant effort.

## PUBLICATIONS RESULTING FROM GRANT EFFORT

### Journal Articles:

Perez, V.M., Renaud, J.E., Watson, L.W., "Adaptive Experimental Design for Construction of Response Surface Approximations", AIAA Journal. (in press).

Tappeta, R.V., Renaud, J.E., Rodriguez, J.F., 2002, "An Interactive Multiobjective Optimization Design Strategy for Decision Based Multidisciplinary Design", *Engineering Optimization*, Volume 34(5), pp.523-544.

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Perez, V.M., Renaud, J.E., Gano, S.E., "Constructing Variable Fidelity Response Surface Approximations in the Usable Feasible Region", *Structural Optimization*, Published by

Springer-Verlag, Germany. (submitted for review)

Gu, X., Renaud, J.E., "Implicit Uncertainty Propagation for Robust Collaborative Optimization", *ASME Journal of Mechanical Design*. (submitted for review)

### Conference Proceedings:

Perez, V.M., Renaud, J.E., Watson, L.M., 2002, "An Interior Point Sequential Approximate Optimization Methodology", AIAA-2002-5505, *Proceedings of the 9th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis & Optimization*, Atlanta, Georgia, September 4-6.

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*Structures, Structural Dynamics, and Materials Conference*, ISBN# 1-56347-560-X, pp. 1-10, Denver, CO, April 22 - 25.

Perez, V.M., Renaud, J.E., Watson, L.M., 2002, "Reduced Sampling for Construction of Quadratic Response Surface Approximations Using Adaptive Experimental Design" AIAA-2002-1587, *CD-Rom Proceedings of the 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, ISBN# 1-56347-560-X, pp. 1-10, Denver, CO, April 22 - 25.

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## STUDENTS SUPPORTED

The following students have been partially supported by this grant effort.

### Doctoral Students

Dr. Ravindra Tappetta, January, 2000 (Eli J. and Helen Shaheen Graduate School Award 2000)

Ms. Xiaoyu Gu (May 2003)

Mr. Victor Perez (May 2003)

Mr. Dhanesh Padmanabhan (May 2003)

Mr. Weiyu Liu (May 2003)

Mr. Harish Agarwal

Mr. Alejandro Espinoza

Mr. Shawn Gano

Mr. Andres Tovar

### Masters Students

Mr. Sean Linden, "Carbon Aircraft Brake Optimization", December 2001.

Ms. Angela Schwebke, May 2001

Mr. Mark Day, , May 2001

Ms. Wenjie Lin, May, 2000